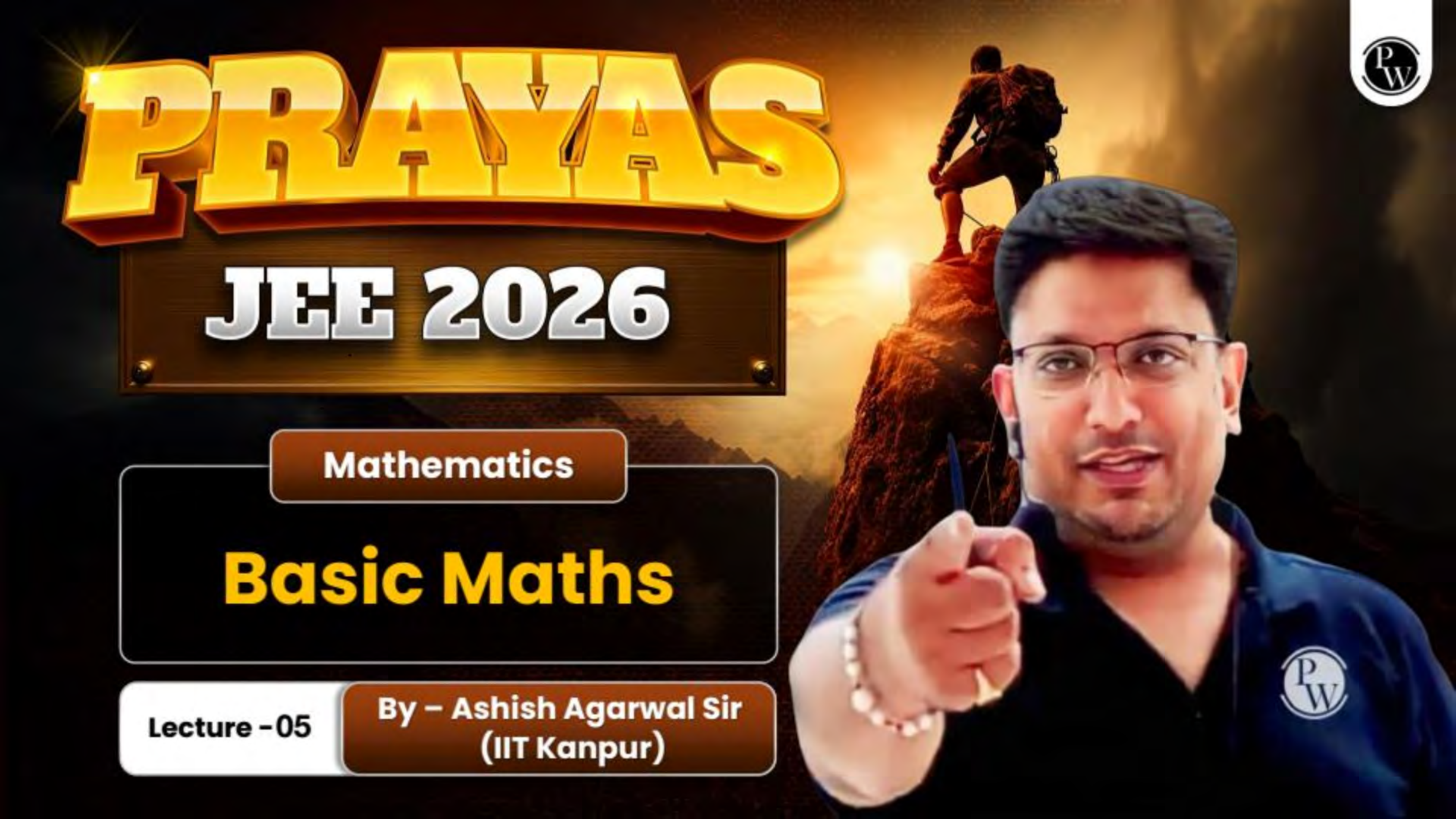

PRAYAS
JEE 2026
Mathematics
Basic Maths
Lecture -05
By – Ashish Agarwal Sir
(IIT Kanpur)

Topics *to be covered*

A Polynomials and their Factorization

B An Important Point

Recap *of previous lecture*

Fill in the Blanks:

1. $x^2 + y^2 + 16z^2 - 4x - 4y + 8z + 9 = 0, x, y, z \in R$ then $x + y + \frac{1}{z} =$ $\underline{\quad 0 \quad}$

$x^2 - 4x + 4 + y^2 - 4y + 4 + (4z)^2 + 2 \cdot 4z \cdot 1 + 1^2 = 0$

$(x-2)^2 + (y-2)^2 + (4z+1)^2 = 0 \Rightarrow x = 2, y = 2, z = -1/4$

2. $(x^2 + 1) + \frac{1}{(x^2+1)}$ has minimum value $\underline{\quad 2 \quad}$ which is attained at $x =$ $\underline{\quad 0 \quad}$

$x^2 + 1 = 1$

$\Downarrow$

$x^2 = 0$

3. $(x^2 - 4)^2 + (y - 2)^4 + (z - \cot \pi/2)^6 = 0$ then $x + y + z$ can be $\underline{\quad 0, 4 \quad}$

$x^2 = 4, \quad y = 2, \quad z = \cot \pi/2 = 0$

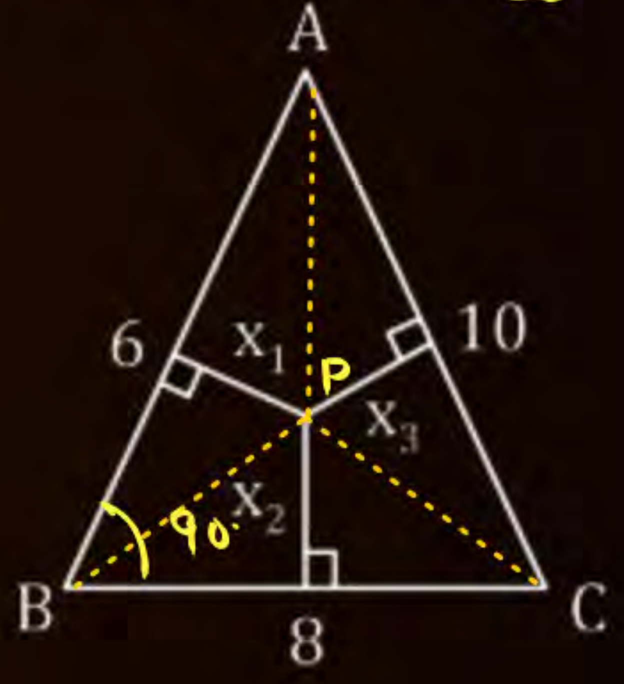
$x = -2, 2$

$x + y + z = 4, 0$

Recap *of previous lecture*

4. The sides of a $\triangle ABC$ are as shown in the figure. Let P be any internal point of this triangle and x_1, x_2 & x_3 denote the distance between P and sides of triangle. The value of $\left(\frac{3x_1+4x_2+5x_3}{6}\right)$ is

$6^2+8^2=10^2 \Rightarrow \angle ABC = \frac{\pi}{2}$



$ar\,\Delta_{BPC} = \frac{1}{2}\cdot 8\cdot x_2$

$ar\,\Delta_{APC} = \frac{1}{2}\cdot x_3\cdot 10$

$ar\,\Delta_{APB} = \frac{1}{2}\cdot 6\cdot x_1$

(+)

$3x_1+4x_2+5x_3$

$\shortparallel$

$ar(\Delta_{ABC})$

$\shortparallel$

$\frac{1}{2}\cdot 6\cdot 8 = 24$

$\frac{3x_1+4x_2+5x_3}{6} = 4.$

Recap *of previous lecture*

5. Perpendicular bisector of chord of a circle passes through it's Centre

6. Area of equilateral triangle of height h ⇒ $\Delta = \frac{h^2}{\sqrt{3}}$

7. $P(x) = x^3 - 6x^2 + 11x - 6$ has $x-1$, $x-2$ & $x-3$ as factors.

$P(1) = 1 - 6 + 11 - 6 = 0 \Rightarrow (x-1)$ is a factor.

$P(2) = 8 - 24 + 22 - 6 = 0 \Rightarrow (x-2)$ is a factor

$P(3) = 27 - 54 + 33 - 6 = 0 \Rightarrow x-3$ is a factor

8. Remainder when a polynomial of degree 5 is divided by a quadratic polynomial is of the form $ax+b$

9. $P(x) = x^4 - 6x^3 + 2x^2 + 2x + 1$ then $P(1) = 1 - 6 + 2 + 2 + 1 = 0$ means $x-1$ is a factor of $P(x)$.

Recap *of previous lecture*

10. Remainder when a polynomial of degree 11 is divided by a cubic polynomial is of the form ax^2+bx+c

11. We say K(x) is a divisor P(x) if remainder = 0

12. Remainder when $P(x) = x^3 - x^2 + 1$ is divided by $2x - 3$ is $P(3/2) = (3/2)^3 - (3/2)^2 + 1$

$P(x) \div ax+b$ Rem $= P\left(-\frac{b}{a}\right)$

$P(x) \div ax-b$ Rem $= P(b/a)$

$= \frac{27}{8} - \frac{9}{4} + 1$

$= \frac{27-18+8}{8} = \underline{17/8}$

Homework Discussion

QUESTION

TAH 02

$4, 7, 10 \ldots$

$T_n = a + (n-1)d$

$= 4 + (n-1)3 = 3n+1$

If $S_n = \frac{1}{\sqrt{1}+\sqrt{4}} + \frac{1}{\sqrt{4}+\sqrt{7}} + \frac{1}{\sqrt{7}+\sqrt{10}} + \ldots$ n terms then -

$$S_n = \frac{1}{\sqrt{4}+\sqrt{1}} + \frac{1}{\sqrt{7}+\sqrt{4}} + \frac{1}{\sqrt{10}+\sqrt{7}} + \cdots + \frac{1}{\sqrt{3n+1}+\sqrt{3n-2}}$$

$$S_n = \frac{1}{3}\left[(\sqrt{4}-1) + (\sqrt{7}-\sqrt{4}) + (\sqrt{10}-\sqrt{7}) + \cdots + (\sqrt{3n+1}-\sqrt{3n-2})\right]$$

$$S_n = \frac{\sqrt{3n+1}-1}{3}$$

(A) $S_8 = \frac{4}{3}$

(B) $S_{16} = 2$

(C) $S_{33} = 3$

(D) $S_{40} = \frac{10}{3}$

Ans. A, B, C, D

QUESTION

TAH 05

If x & y are positive integers, such that $\frac{1}{x}+\frac{1}{y}=\frac{1}{6}$ & $x \geq y$, then the number of ordered pairs of (x, y) is

$$\frac{x+y}{xy}=\frac{1}{6}$$

$$6(x+y)=xy$$

$$6x+6y-xy=0$$

$$6x-36-y(x-6)=0-36$$

$$6(x-6)-y(x-6)=-36$$

$$(x-6)(y-6)=36.$$

$x-6$	$y-6$	(x,y)
6	6	(12,12)
9	4	(15,10)
12	3	(18,9)
36	1	(42,7)
18	2	(24,8)

Ans. 5

Aao Machaay Dhamaal

Deh Swaal pe Deh Swaal

QUESTION

If $(x^3 + ax^2 + bx + 6)$ has $(x - 2)$ as a factor and leaves a remainder 3 when divided by $(x - 3)$

(A) ~~sum of absolute value of $(a + b)$ is 4~~

(B) ~~product of ab is 3~~

(C) ~~difference of a & b is 14~~

(D) ~~a^b is defined~~ $(-3)^{-1} = \frac{1}{(-3)^1} = -\frac{1}{3}$

Since $x-2$ is a factor of $P(x) = x^3+ax^2+bx+6$

$\Rightarrow P(2)=0 \Rightarrow 8+4a+2b+6=0$

$2a+b=-7$ — (i)

Also $P(3)=3$ (By Rem Thm)

$27+9a+3b+6=3$

$9a+3b=-30$

$3a+b=-10$ — (ii)

$\ominus$

$-a=3$

$a=-3$

$b=-1$

(A) $|a+b| = |-3-1| = 4$

Ans. A, B, D

QUESTION

TAH01

If $81x^5 + 27x^3 - 9x^2 + 50$ is divided by $(3x + 2)$ then remainder is λ then

(A) $\frac{3\lambda - 42}{10}$ is equal to 4

(B) if $\lambda = \frac{p}{q}$ then $(p + q)$ is divisible by 17 (where p & q are coprime)

(C) λ is a natural number

(D) $\left(\lambda - \frac{1}{3}\right)$ is divisible by 3

Ans. A, B, D

QUESTION

Suppose p(x) is a polynomial with integer coefficients. The remainder when p(x) is divided by x – 1 is 1 and the remainder when p(x) is divided by x – 4 is 10. If r(x) is the remainder when p(x) is divided by (x – 1) (x – 4), find the value of r(2006).

$P(x) \div (x-1)$ Rem $=1$ — $P(1)=1$

$P(x) \div (x-4)$ Rem $=10$ — $P(4)=10$

$P(x) \div (x-1)(x-4)$ Rem $= r(x)$

Rem theorem is applicable for linear Divisors

$P(x) = Q(x)\cdot \underbrace{(x-1)(x-4)}_{\text{degree 2}} + (ax+b)$, i.e. $r(x) = ax+b$.

Now put $x=1$ $P(1) = a+b = 1$

put $x=4$ $P(4) = 4a+b = 10$

$3a = 9$

$a = 3$

$b = -2$

$r(x) = 3x-2$

$r(2006) = 6018 - 2 = 6016$

Ans. 6016

QUESTION

Tah 02

Let $f(x) = x^{135} + x^{125} - x^{115} + x^5 + 1$. If $f(x)$ is divided by $x^3 - x$ then the remainder is some function of x say $g(x)$. Find the value of $g(10)$.

Ans. 21

QUESTION

Polynomial $P(x)$ contains only terms of odd degree. When $P(x)$ is divided by $(x-3)$ the remainder is 6. If $P(x)$ is divided by (x^2-9) then remainder is $g(x)$. Find value of $g(2)$.

$P(x)$ — contains terms of only odd degree

Ex: $P(x) = x^7 - 6x^3 + 5x$
$P(x) = x^9 + 5x^3 - 2x$

observe!! $P(-x) = -P(x)$

$P(-3) = -P(3)$
$= -6$

$P(-x) = -x^7 + 6x^3 - 5x$
$= -(x^7 - 6x^3 + 5x)$
$= -P(x)$

Now

$P(x) \div x-3$ — Rem $= 6 = P(3)$

$P(x) \div x^2-9$ — Rem $= ?$

$P(x) = (x^2-9)Q(x) + ax + b$, $g(x) = ax+b$

$P(3) = 0 + 3a + b = 6$
$P(-3) = 0 - 3a + b = -6.$

$2b = 0 \rightarrow b = 0$

$a = 2$

$g(x) = 2x$
$g(2) = 4$

Ans. 4

Factors of a Polynomial

$(x-1)^2(x-2)=0$

roots 1, 1, 2

soln : 1, 2

- $ax^2+bx+c \begin{cases} \alpha \\ \beta \end{cases} \Rightarrow ax^2+bx+c=a(x-\alpha)(x-\beta)$

- $ax^3+bx^2+cx+d \begin{cases} \alpha \\ \beta \\ \gamma \end{cases} \Rightarrow ax^3+bx^2+cx+d=a(x-\alpha)(x-\beta)(x-\gamma)$

- $ax^4+bx^3+cx^2+dx+e \begin{cases} \alpha \\ \beta \\ \gamma \\ \delta \end{cases} \Rightarrow ax^4+bx^3+cx^2+dx+e$
$=a(x-\alpha)(x-\beta)(x-\gamma)(x-\delta)$

QUESTION

Find three degree polynomial P(x) whose zeroes are $\frac{-1}{3}, \frac{-2}{3}$, 4 and P(1) =1

$P(x) \begin{cases} -\frac{1}{3} \\ -\frac{2}{3} \\ 4 \end{cases}$

$\downarrow$

degree

$$P(x) = a \cdot \left(x - \left(-\frac{1}{3}\right)\right)\left(x - \left(-\frac{2}{3}\right)\right)(x-4)$$

$$P(x) = a\left(x+\frac{1}{3}\right)(x+2/3)(x-4)$$

$$P(1) = a(4/3)(5/3)(-3) = 1$$

$$a\left(\frac{-20}{3}\right) = 1$$

$$a = -3/20$$

$$P(x) = -\frac{3}{20}(x+1/3)(x+2/3)(x-4)$$

QUESTION

★★★KCLS★★★

If 4 degree polynomial P(x) is such that P(1) = 1, P(2) = 2, P(3) = 3 and P(4) = 4. Then find P(5) (leading coefficient is 1).

$g(x) = \underbrace{x^4 + (\ )x^3 + (\ )x^2 + (\ )x + c}_{P(x)} - x$

degree $g(x) = 4$

leading coeff $= 1$

consider $g(x) = P(x) - x$

$g(1) = P(1) - 1 = 0$

$g(2) = P(2) - 2 = 0$

$g(3) = P(3) - 3 = 0$

$g(4) = P(4) - 4 = 0$

$g(x)$ — 1, 2, 3, 4

$g(x) = 1 \cdot (x-1)(x-2)(x-3)(x-4)$

$P(x) - x = (x-1)(x-2)(x-3)(x-4)$

$P(x) = (x-1)(x-2)(x-3)(x-4) + x$

$P(5) = (5-1)(5-2)(5-3)(5-4) + 5$

$= 4! + 5 = 29$

Concept : consider

$g(x) = P(x) -$ Jo apko lagay P(x) hai

Mazdoori naa kare

$P(x) = ax^4 + bx^3 + cx^2 + dx + e$

$P(1) = 1 \Rightarrow a + b + c + d + e = 4$

$P(2) = 2 \Rightarrow 16a + 8b + 4c + 2d + e = 2$

⋮

QUESTION

Tan 03

If p(x) is a polynomial of 3 degree for which p(1) = 1, p(2) = 4, p(3) = 9 then find the value of p(5) and leading coefficient be 2.

$$g(x) = p(x) - x^2$$

Ans. 73

QUESTION

2. $P(1)=1$ 3. $P(2)=2$ 4. $P(3)=3$ 5. $P(4)=4$ 6 $P(5)=6$

★★★KCLS★★★

There is a 4 degree polynomial p(x) with $P(1) = \frac{1}{2}, P(2) = \frac{2}{3}, P(3) = \frac{3}{4}, P(4) = \frac{4}{5}$ and $P(5) = \frac{5}{6}$ then find P(6).

Consider $g(x) = h(x) - x \implies g(x) = (x+1)P(x) - x$

$g(1) = 2 \cdot P(1) - 1 = 0$

g(x) degree = 5

$g(2)=0,\ g(3)=0,\ g(4)=0,\ g(5)=0$

$g(x) \to 1, 2, 3, 4, 5$

$g(x) = a(x-1)(x-2)(x-3)(x-4)(x-5)$

$(x+1)\cdot P(x) - x = a(x-1)(x-2)(x-3)(x-4)(x-5)$

$P(x) = \frac{a(x-1)(x-2) - - (x-5) + x}{(x+1)}$ But P(x) is a polynomial

lagtaa hai !!

$P(x) = \frac{x}{x+1}$

$\underbrace{(x+1)P(x) = x}$

$\cancel{g(x) = P(x) - \frac{x}{x+1}}$

Not a polynomial

let $(x+1)P(x) = h(x)$

$h(x) = x$

Ans. 5/6

$R(x) = a(x-1)(x-2) \cdots (x-5) + x$ should have $x+1$ as a factor.

$\Downarrow$

$R(-1) = 0 \longrightarrow a(-2)(-3)(-4)(-5)(-6) - 1 = 0$

$a = \frac{-1}{720}$

$$P(x) = \frac{a(x-1)(x-2)(x-3)(x-4)(x-5) + x}{1+x}$$

$$P(6) = \frac{\frac{-1}{720} \cdot 120 + 6}{7} = \frac{-\frac{1}{6} + 6}{7} = \frac{35}{7 \times 6} = 5/6 \text{ Ans}$$

A Golden Point

$E = a^2 + b^2 + c^2 - ab - bc - ca$ (where $a, b, c \in R$)

$$E = \frac{1}{2}\left(2a^2 + 2b^2 + 2c^2 - 2ab - 2bc - 2ca\right)$$

$$= \frac{1}{2}\left(a^2 + c^2 - 2ac + a^2 + b^2 - 2ab + b^2 + c^2 - 2bc\right)$$

$$= \frac{1}{2}\left(\underbrace{(a-c)^2}_{\geq 0} + \underbrace{(a-b)^2}_{\geq 0} + \underbrace{(b-c)^2}_{\geq 0}\right) \geq 0$$

① $E = a^2 + b^2 + c^2 - ab - bc - ca \geq 0$

② $a^2 + b^2 + c^2 - ab - bc - ca = 0 \Leftrightarrow a = b = c$

QUESTION

If a, b, c denotes the length of sides of the triangle such that
$9a^2 + 16b^2 + 25c^2 - 12ab - 20bc - 15ac = 0$ then ratio $a : b : c$ is-

(A) $20 : 15 : 12$

(B) $3 : 4 : 5$

(C) $2 : 1 : 2$

(D) $1 : 2 : 3$

$$(3a)^2 + (4b)^2 + (5c)^2 - 3a \cdot 4b - 4b \cdot 5c - 5c \cdot 3a = 0$$

$$3a = 4b = 5c = 60\lambda$$

$$a = 20\lambda$$
$$b = 15\lambda$$
$$c = 12\lambda$$

$$a : b : c = 20\lambda : 15\lambda : 12\lambda$$
$$= 20 : 15 : 12$$

Ans. A

QUESTION

★★★ KCLS ★★★

Find all real numbers x satisfying's $2^x + 3^x - 4^x + 6^x - 9^x = 1$.

$$4^x + 9^x + 1 - 2^x - 3^x - 6^x = 0$$

$$(2^x)^2 + (3^x)^2 + (1^x)^2 - 2^x \cdot 3^x - 3^x \cdot 1^x - 2^x \cdot 1^x = 0$$

$$2^x = 3^x = 1^x$$

$$2^x = 3^x = 1$$

$x = 0$ Ans

Saari Class Illustrations
Retry karni Hai

NO HOMEWORK TODAY!!

ENJOY LIFE....

Solution to Previous TAH

QUESTION

TAH 01

If $x = \sqrt{33 - 20\sqrt{2}}$ & $y = \sqrt{54 - 20\sqrt{2}}$ then value of $x - y$ is equal to

(A) $3(1 + \sqrt{2})$

(B) $7(\sqrt{2} - 1)$

(C) $\dfrac{-7}{1 + \sqrt{2}}$

(D) $7(1 + \sqrt{2})$

Ans. C

Q

$x = \sqrt{33 - 20\sqrt{2}}$ $y = \sqrt{54 - 20\sqrt{2}}$ then $x - y = ?$ (Jah - 01)

$x = \sqrt{(5)^2 + (2\sqrt{2})^2 - 2 \times 5 \times 2\sqrt{2}}$

$x = \sqrt{(5 - 2\sqrt{2})^2}$

$x = |\underbrace{5 - 2\sqrt{2}}_{+ve}|$

$y = \sqrt{(5\sqrt{2})^2 + (2)^2 - 2 \cdot 5\sqrt{2} \cdot 2}$

$y = \sqrt{(5\sqrt{2} - 2)^2}$

$y = \sqrt{(2 - 5\sqrt{2})^2}$

$y = |\underbrace{2 - 5\sqrt{2}}_{-ve}|$

$x - y = (5 - 2\sqrt{2}) - (-(2 - 5\sqrt{2}))$

$x - y = 5 - 2\sqrt{2} + 2 - 5\sqrt{2} = 7 - 7\sqrt{2} = 7(1 - \sqrt{2})$

$x - y = \dfrac{-7(\sqrt{2} - 1) \times (\sqrt{2} + 1)}{(\sqrt{2} + 1)} = \dfrac{-7(2 - 1)}{(\sqrt{2} + 1)} = \dfrac{-7}{(\sqrt{2} + 1)}$

$\therefore x - y = \dfrac{-7}{(\sqrt{2} + 1)}$ Ans

AKASH KUMAR
SIWAN, BIHAR

L-04

TAH-1

→ $x = \sqrt{33 - 20\sqrt{2}}$ & $y = \sqrt{54 - 20\sqrt{2}}$ then value of $x - y$ is equal to.

Sol^n

$$x - y = \sqrt{33 - 20\sqrt{2}} - \sqrt{54 - 20\sqrt{2}}$$

$$= \sqrt{33 - 2\times10\times\sqrt{2}} - \sqrt{54 - 2\times10\times\sqrt{2}}$$

$$= \sqrt{33 - 2\times5\times2\sqrt{2}} - \sqrt{54 - 2\times10\sqrt{2}}$$

$$= \sqrt{(5)^2 + (\sqrt{8})^2 - 2\times5\times\sqrt{8}} - \sqrt{54 - 2\times10\times\sqrt{2}}$$

$$= \sqrt{(5 - \sqrt{8})^2} - \sqrt{54 - 2\times2\times5\sqrt{2}}$$

$$= |5 - \sqrt{8}| - \sqrt{(2)^2 + (\sqrt{50})^2 - 2\times2\times\sqrt{50}}$$

$$= 5 - \sqrt{8} - \sqrt{(\sqrt{50} - 2)^2}$$

$$= 5 - \sqrt{8} - (|\sqrt{50} - 2|)$$

$$= 5 - \sqrt{8} - \sqrt{50} + 2$$

$$= 7 - 2\sqrt{2} - 5\sqrt{2}$$

$$= 7(1 - \sqrt{2}) \times \frac{(1+\sqrt{2})}{(1+\sqrt{2})} = \frac{7(1-2)}{1+\sqrt{2}} = \frac{-7}{1+\sqrt{2}}$$ // Ans.

(c)

$2\times5\times2\sqrt{2}$
$= 2\times5\sqrt{8}$
$(\sqrt{33})^2$
$25 + 8 = 33$

$2\times2\times5\times\sqrt{2}$
$2\times2\times\sqrt{50}$
$25 + 72$

QUESTION

TAH 02

If $S_n = \frac{1}{\sqrt{1}+\sqrt{4}} + \frac{1}{\sqrt{4}+\sqrt{7}} + \frac{1}{\sqrt{7}+\sqrt{10}} +$ ..... n terms then -

(A) $S_8 = \frac{4}{3}$

(B) $S_{16} = 2$

(C) $S_{33} = 3$

(D) $S_{40} = \frac{10}{3}$

Ans. A, B, C, D

TAh: (2) $S_n = \frac{1}{\sqrt{1}+\sqrt{4}} + \frac{1}{\sqrt{4}+\sqrt{7}} + \frac{1}{\sqrt{7}+\sqrt{10}} + \cdots + n$ term

(A) $S_8 = 4/3$ (B) $S_{16} = 2$ (C) $S_{33} = 3$ (D) $S_{40} = 10/3$

Sol. $1, 4, 7, \cdots n \rightarrow T_n = 1+(n-1)3 = 3n-2$

$$S_n = \frac{1}{\sqrt{1}+\sqrt{4}} + \frac{1}{\sqrt{4}+\sqrt{7}} + \frac{1}{\sqrt{7}+\sqrt{10}} + \cdots + \frac{1}{\sqrt{3n-2}+\sqrt{3n+1}}$$

$$S_n = \frac{\sqrt{4}-\sqrt{1}}{4-1} + \frac{\sqrt{7}-\sqrt{4}}{7-4} + \frac{\sqrt{10}-\sqrt{7}}{10-7} + \cdots + \frac{\sqrt{3n+1}-\sqrt{3n-2}}{(3n+1)-(3n-2)}$$

$$S_n = \frac{(\cancel{\sqrt{4}}-\sqrt{1}) + (\cancel{\sqrt{7}}-\cancel{\sqrt{4}}) + (\cancel{\sqrt{10}}-\cancel{\sqrt{7}}) + \cdots + (\sqrt{3n+1}-\cancel{\sqrt{3n-2}})}{3}$$

$$= \frac{\sqrt{3n+1}-1}{3}$$

24/04/2025
Himanshu Ramuka.
S.O.: Sushil Ramuka.
Rajasthan.

(A) $S_8 = \frac{\sqrt{8(3)+1}-1}{3} = 4/3$

(B) $S_{16} = \frac{\sqrt{3(16)+1}-1}{3} = 6/2 = 3$

(C) $S_{33} = \frac{\sqrt{3(33)+1}-1}{3} = 9/3 = 3$

(D) $S_{40} = \frac{\sqrt{3(40)+1}-1}{3} = \frac{\sqrt{121}-1}{3}$

$= \frac{11-1}{3} = 10/3$

QUESTION

TAH 03

If $x = \sqrt{2+\sqrt{3}} + \sqrt{4-\sqrt{15}}$ then value of $\sqrt{2}x$ is equal to

(A) $\sqrt{5}-\sqrt{3}$

(B) $\sqrt{5}-1$

(C) $\sqrt{3}+\sqrt{5}$

(D) $\sqrt{5}+1$

Ans. D

TA 3 $\quad x = \sqrt{2+\sqrt{3}} + \sqrt{4-\sqrt{15}}$ then $\sqrt{2}x = ?$

Sol^n $\quad \sqrt{2}x = \sqrt{2}\sqrt{2+\sqrt{3}} + \sqrt{2}\sqrt{4-\sqrt{15}}$

$= \sqrt{4+2\sqrt{3}} + \sqrt{8-2\sqrt{15}}$

$= \sqrt{(\sqrt{3}+1)^2} + \sqrt{(\sqrt{5}-\sqrt{3})^2}$

$= |\sqrt{3}+1| + |\sqrt{5}-\sqrt{3}|$

$= \cancel{\sqrt{3}}+1 + \sqrt{5} - \cancel{\sqrt{3}}$

$= 1+\sqrt{5}$ Ans

QUESTION

TAH 04

If $a = \sqrt{6+2\sqrt{5}} - \sqrt{6-2\sqrt{5}}$; $b = \sqrt[3]{6\sqrt{3}+10} + \sqrt[3]{10-6\sqrt{3}}$, then the value of (ab) is equal to

(A) 8

(B) 12

(C) 4

(D) 6

Ans. C

Q. If $a=\sqrt{6+2\sqrt{5}}-\sqrt{6-2\sqrt{5}}$; $b=\sqrt[3]{6\sqrt{3}+10}+\sqrt[3]{10-6\sqrt{3}}$, then the value of (ab) is equal to — Tah-04

Sol

$$a=\sqrt{(\sqrt{5})^2+(1)^2+2\sqrt{5}}-\sqrt{(\sqrt{5})^2+(1)^2-2\sqrt{5}}$$

$$a=\sqrt{(\sqrt{5}+1)^2}-\sqrt{(\sqrt{5}-1)^2}$$

$$a=|\sqrt{5}+1|-|\sqrt{5}-1|=\sqrt{5}+1-(\sqrt{5}-1)$$

$$a=\cancel{\sqrt{5}}+1-\cancel{\sqrt{5}}+1=2$$

$$b=\sqrt[3]{6\sqrt{3}+10}+\sqrt[3]{10-6\sqrt{3}}$$

C.BS

$$b^3=(6\sqrt{3}+10)+(10-6\sqrt{3})+3\sqrt[3]{6\sqrt{3}+10}\,\sqrt[3]{10-6\sqrt{3}}\,(\underbrace{\sqrt[3]{6\sqrt{3}+10}+\sqrt[3]{10-6\sqrt{3}}}_{b})$$

$$b^3=6\sqrt{3}+20-6\sqrt{3}+3\sqrt[3]{100-108}\cdot b$$

$$b^3=\cancel{6\sqrt{3}}+20-\cancel{6\sqrt{3}}+3\sqrt[3]{(-2)^3}\cdot b$$

$$b^3=20-6b$$

$b^3 + 6b - 20 = 0 \longrightarrow (b-2)(b^2 + 2b + 10) = 0$

$b = 2$ or $b^2 + 2b + 10 = 0$

$D = 4 - 10 \cdot 4 < 0$

$\downarrow$

No real roots

$P(b) = b^3 + 6b - 20$

$P(2) = 8 + 12 - 20 = 0$

$(b-2)$ is a factor

$b(b^2 + 6) = 20$

$b = 20$ or $b^2 + 6 = 20$

Gadho | Gadhiyoo aisaa naa karo

QUESTION

TAH 05

If x & y are positive integers, such that $\frac{1}{x}+\frac{1}{y}=\frac{1}{6}$ & $x \geq y$, then the number of ordered pairs of (x, y) is

Ans. 5

~~TAH5~~

Q If x & y are positive integers, such that $\frac{1}{x} + \frac{1}{y} = \frac{1}{6}$ & $x \geqslant y$, then the number of ordered pairs of (x, y) is .

Solⁿ

$$\frac{1}{x} + \frac{1}{y} = \frac{1}{6}$$

$$\frac{x+y}{xy} = \frac{1}{6}$$

$$6x + 6y = xy$$

$$6x + 6y - xy = 0$$

$$6x - y(-6 + x) + 36 - 36 = 0$$

$$6x - 36 - y(x-6) + 36 = 0$$

$$6(x-6) - y(x-6) + 36 = 0$$

$$(x-6)(6-y) + 36 = 0$$

$$(x-6)(y-6) = 36$$

36

1×36	36×1	2×18	18×2	4×9	9×4	6×6	3×12	12×3
$(7, 42)$	$(42, 7)$	$(8, 24)$	$(24, 8)$	$(10, 15)$	$(15, 9)$	$(12, 12)$	$(9, 18)$	$(18, 9)$

ATQ $x, y \in I^{+}$ & $x \geqslant y$

$\therefore$ final pairs : $(42, 7)$, $(24, 8)$, $(15, 9)$, $(18, 9)$
$(12, 12)$

$\therefore$ n(ordered pairs) = ~~4~~ 5 Ans

Solution to Previous KTKs

QUESTION

(KTK 1)

The expression $\sqrt{12+6\sqrt{3}}+\sqrt{12-6\sqrt{3}}$ simplifies to

(A) 4

(B) $2\sqrt{3}$

(C) $3\sqrt{3}$

(D) 6

Ans. D

Lecture 4 24 April 2025

KTK 1 The expression $\sqrt{12+6\sqrt{3}} + \sqrt{12-6\sqrt{3}}$ simplifies to

Soln Let $x = \sqrt{12+6\sqrt{3}} + \sqrt{12-6\sqrt{3}}$

$$x = \sqrt{(3+\sqrt{3})^2} + \sqrt{(3-\sqrt{3})^2}$$

$$x = |3+\sqrt{3}| + |3-\sqrt{3}|$$

$$x = 3+\cancel{\sqrt{3}} + 3 - \cancel{\sqrt{3}}$$

$$x = \underline{6}$$ Ans

QUESTION

(KTK 2)

Let p, q be real numbers satisfying $p^2 - q^2 = 4$ and $2pq = 3$ then $(p^2 + q^2)$ is equal to

(A) 1

(B) 9

(C) 16

(D) 5

Ans. D

① Let p, q be real nos satisfying $p^2 - q^2 = 4$ and $2pq = 3$ then $(p^2 + q^2)$ is equal to.

KTK-02

Sol

$$p^2 - q^2 = 4 \text{ SBS}$$

$$\{(p+q)(p-q)\}^2 = (4)^2$$

$$(p+q)^2 (p-q)^2 = 16$$

$$(p^2+q^2+2pq)(p^2+q^2-2pq) = 16$$

$$(p^2+q^2)^2 - (2pq)^2 = 16$$

$$(p^2+q^2)^2 - (3)^2 = 16$$

$$(p^2+q^2)^2 = 16 + 9$$

$$(p^2+q^2) = \sqrt{25} = 5$$

$$\boxed{p^2+q^2 = 5} \text{ Ay}$$

AKASH KUMAR
SIWAN, BIHAR

KTK-03

QUESTION

(KTK 3)

Value of x satisfying the equation $\sqrt{x^2 + 2x - 63} + |x^2 - 9x + 14| = 0$ is

KTK 3 Value of x satisfying the equation $\sqrt{x^2+2x-63} + |x^2-9x+14| = 0$ is

Sol^n $\underbrace{\sqrt{x^2+2x-63}}_{(\geq 0)} + \underbrace{|x^2-9x+14|}_{(\geq 0)} = 0$

$$\Rightarrow \quad x^2+2x-63=0 \qquad\qquad x^2-9x+14=0$$

$$(x-7)(x+9)=0 \qquad\qquad (x-7)(x-2)=0$$

$$x = 7, -9 \qquad\qquad x = 2, 7$$

$$\cap$$

$$\boxed{x = 7}$$ Ans

QUESTION

(KTK 4)

The expression $\sqrt{(28+10\sqrt{3})}+\sqrt{(28-10\sqrt{3})}$ simplifies to

A 10

B 12

C $2\sqrt{3}$

D 5

Ans. A

KTK-4

The expression $\sqrt{(28+10\sqrt{3})}+\sqrt{(28-10\sqrt{3})}$ simplifies to

(A) 10 (B) 12 (C) $2\sqrt{3}$ (D) 5

$\Rightarrow \sqrt{(28+10\sqrt{3})}+\sqrt{(28-10\sqrt{3})}$

$= \sqrt{25+3+2\times5\times\sqrt{3}}+\sqrt{25+3-2\times5\times\sqrt{3}}$

$= \sqrt{(5+\sqrt{3})^2}+\sqrt{(5-\sqrt{3})^2}$

$= \left|\underset{(+ve)}{5+\sqrt{3}}\right| + \left|\underset{(+ve)}{5-\sqrt{3}}\right|$

$= 5+\cancel{\sqrt{3}}+5-\cancel{\sqrt{3}} = 10$ —— (A) (Ans)

(9)

KTK The expression $\sqrt{(28+10\sqrt{3})}+\sqrt{28-10\sqrt{3}}$ simplifies to

Soln Let $X = \sqrt{28+10\sqrt{3}} + \sqrt{28-10\sqrt{3}}$

$$= \sqrt{(5+\sqrt{3})^2} + \sqrt{(5-\sqrt{3})^2}$$

$$= |5+\sqrt{3}| + |5-\sqrt{3}|$$

$$= 5+\cancel{\sqrt{3}}+5-\cancel{\sqrt{3}}$$

$$= 10$$

= Ans

QUESTION

(KTK 5)

Find all the integral solutions of the equation $xy = 2x - y$.

Ans. $(0, 0), (-2, 4), (1, 1), (-3, 3)$

KTK 5 Find all integral solutions of eqⁿ $xy = 2x - y$

Solⁿ

$$xy = 2x - y$$
$$2x - y - xy = 0$$
$$2x - y(1+x) + 2 - 2 = 0 \qquad \text{[Simon factorisation]}$$
$$2x + 2 - y(1+x) - 2 = 0$$
$$2(x+1) - y(x+1) - 2 = 0$$
$$(x+1)(2-y) - 2 = 0$$
$$(x+1)(y-2) = -2$$

1×-2	-2×1	-1×2	2×-1
$(0,0)$	$(-3,3)$	$(-2,4)$	$(1,1)$

All solⁿ are $(0,0),\ (1,1),\ (-2,4),\ (-3,3)$

KTK - 5

Find all the integral solutions of eqn. $xy = 2x - y$.

$\Rightarrow 2x - y = xy$

$\Rightarrow 2x - y - xy = 0$

$\Rightarrow 2x - y(1 + x) = 0$

$\Rightarrow 2 - 2 + 2x - y(1 + x) = 0$

$\Rightarrow -2 + 2(1 + x) - y(1 + x) = 0$

$\Rightarrow -2 + (2 - y)(1 + x) = 0$

$\Rightarrow (2 - y)(1 + x) = 2$

$$\left.\begin{array}{l} 1 \times 2 \longrightarrow (x, y) = (1, 1) \\ 2 \times 1 \longrightarrow (x, y) = (0, 0) \\ -1 \times -2 \longrightarrow (x, y) = (-3, 3) \\ -2 \times -1 \longrightarrow (x, y) = (-2, 4) \end{array}\right\} \text{(Ans)}$$

Q Find the integral solutions of the equation

Sol $xy = 2x - y$ (KTK-5)

$$xy - 2x = -y$$

$$x(y-2) = -y$$

$$x = \frac{-y+2-2}{y-2}$$

$$x = \frac{-(y-2)-2}{(y-2)}$$

$$x = -1 - \frac{2}{y-2}$$

$y-2$ should be a divisor of 2

$$y - 2 = -2, -1, 1, 2$$

$$y = 0, 1, 3, 4$$

$$x = 0, 1, -3, -2$$

$$\therefore (x, y) = (0,0)\ (1,1)\ (-3,3)\ (-2,4)$$

4 pairs

AKASH KUMAR
SIWAN, BIHAR

Page No.

Date

K (5): Find all the integral solutions of eq^n

$$xy = 2x - y$$

Sol. $\therefore 2x - xy - y = 0$

$\Rightarrow x(2-y) - y + 2 - 2 = 0$

$\Rightarrow x(2-y) + 2 - y - 2 = 0$

$\Rightarrow (2-y)(x+1) = 2$

1	2	$\longrightarrow (x, y) \equiv (1, 1)$
2	1	$\longrightarrow (x, y) \equiv (0, 0)$
-1	-2	$\longrightarrow (x, y) \equiv (-3, 3)$
-2	-1	$\longrightarrow (x, y) \equiv (-2, 4)$

24/04/2025
Himanshu Ramuka.
S.O.: Sushil Ramuka.
Rajasthan.

THANK YOU